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The Dielectric Boundary Condition for the Embedded Curved Boundary (ECB) Method

D. W. Hewett
C. S. Kueny

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Embedded Curved Boundary (ECB) Method

Dennis W. Hewett and Christopher S. Kueny
D&NT, LLNL, Livermore CA 94551

A new version of ECB[1] has been completed that allows nonuniform grid spacing and a new dielectric boundary condition. ECB was developed to retain the simplicity and speed of an orthogonal mesh while capturing much of the fidelity of adaptive, unstructured finite element meshes. Codes based on orthogonal meshes are easy to work with and lead to well-posed elliptic and parabolic problems that are comparatively easy to solve. Generally, othogonal mesh representations lead to banded matrices while unstructured representations lead to more complicated sparse matrices. Recent advances in adapting banded linear systems to massively parallel computers reinforce our opinion that iterative field solutions utilizing banded matrix methods will continue to be competitive[2]. Unfortunately, the underlying "stair-step" boundary representation in simple orthogonal mesh (and recent Adaptive Mesh Refinement) applications is inadequate. With ECB, the curved boundary is represented by piece-wise-linear representations of curved internal boundaries embedded into the orthogonal mesh—-we build better, but not more, coefficients in the vicinity of these boundaries–and we use the surplus free energy on more ambitious physics models.

ECB structures are constructed out of the superposition of analytically prescribed building blocks. In 2-D, we presently use a POLY4 (linear boundaries defined by 4 end points), an ANNULUS, (center, inner & outer radii, starting & stopping angle), a ROUND (starting point & angle, stopping point & angle, fillet radius). A link-list AIRFOIL has also been constructed. In the ECB scheme, we first find each intercept of the structure boundary with an I or J grid line is assigned an index K. We store the actual x, y value at the intercept, and the slope of the boundary at that intercept, in arrays whose index K is associated with the corresponding mesh point just inside the structure. In 2-D, a point just outside a structure may have up to 4 intercepts associated with it. We now construct PieceWise Linear Segments (PWLS)–from the slope and intercept– that will serve as the computational boundary–a boundary constructed as a superposition of the analytic elements given as input.

The intersection of these PWLSs with other nearby PWLSs define the endpoints. We insist that there be an endpoint within each cell for each sequence of PWLSs coming through that cell. We give directionality to a PWLS by the convention that looking from point 1 to point 2 will place the interior of the structure on the left. This scheme allows the generalization to those boundary structures that do not actually have any mesh points inside a structure[1]. Since these structures are defined by their analytically defined bounding walls, points can always be found that are "inside" a given wall segment–even though it may, in fact, be outside the opposite bounding segment. Such subgrid geometry is useful, for example, in a gun configuration with a thin electrode, perhaps not parallel to any axes of the orthogonal mesh, in which the electrode simply contributes to an external focusing field.

ECB Coefficients for Elliptic Equations

For simplicity, we consider a 1D Poisson equation. The equation is

$$\frac{\partial\, \epsilon\, \partial S}{\partial x^2} = \frac{\epsilon_p \frac{S_{i+1}-S}{\Delta_p} - \epsilon_m \frac{S-S_{i-1}}{\Delta_m}}{(\Delta_m + \Delta_p)/2} = -\rho$$

where $\Delta_m{=}x_i - x_{i-1}$, $\Delta_p{=}x_{i+1} - x_i$, and $\epsilon_m{=}(\epsilon_i + \epsilon_{i-1})/2$, $\epsilon_p{=}(\epsilon_{i+1} + \epsilon_i)/2$. Defining $\Delta \equiv (\Delta_m + \Delta_p)/2$,

$$\epsilon_p S_p/\Delta_p\Delta - S\left[\epsilon_p/\Delta_p + \epsilon_m/\Delta_m\right]/\Delta + \epsilon_m S_m/\Delta_m\Delta = -\rho\ , \tag{1}$$

we first form and store coefficients for all points I as if there were no boundaries, then redefine "smarter" coefficients that describe the boundary constraints at those points just outside of a structure.

Dirichlet Boundary Conditions

Consider a location i just outside of (below) a structure intercept

Δ_m Δ_p

|————————————|——————/—|

S_m S δ_p S_i S_p

For Dirichlet boundary conditions, we specify the value B at that intercept S_i by using (1) as if it had a closer mesh point for the $i+1$ location

$$S_i = B, \qquad \epsilon(B - S)/\delta_p\Delta - \epsilon_m(S - S_m)/\Delta_m\Delta = -\rho$$

The coefficients for a tridiagonal matrix at points i and $i+1$ (and all other interior points) are

$$\begin{array}{rcc} & i & i+1, interior \\ & \text{---------} & \text{---------} \\ S_p: & 0 & 0 \\ S & (\epsilon_m/\Delta_m\Delta + \epsilon/\delta_p\Delta) & 1 \\ S_m: & \epsilon_m/\Delta_m\Delta & 0 \\ rhs: & -\rho - \epsilon B/\delta_p\Delta & B \end{array} \tag{2}$$

Dielectric Boundary Conditions

Consider again the location i just outside of (below) a structure intercept. We wish to impose the first-order dielectric condition

$$\left|\epsilon\frac{\partial S}{\partial x}\right|_1^2 = \epsilon_{p1}\frac{S_p - S_i}{\Delta_p - \delta_p} - \epsilon\frac{S_i - S}{\delta_p} = \sigma \tag{3}$$

where ϵ_{p1}, ϵ are evaluated at points $i+1$ and i. We now solve for S_i and use it with (1), again as if it had a shorter leg in the templet. Solving for S_i and plugging into (1) gives

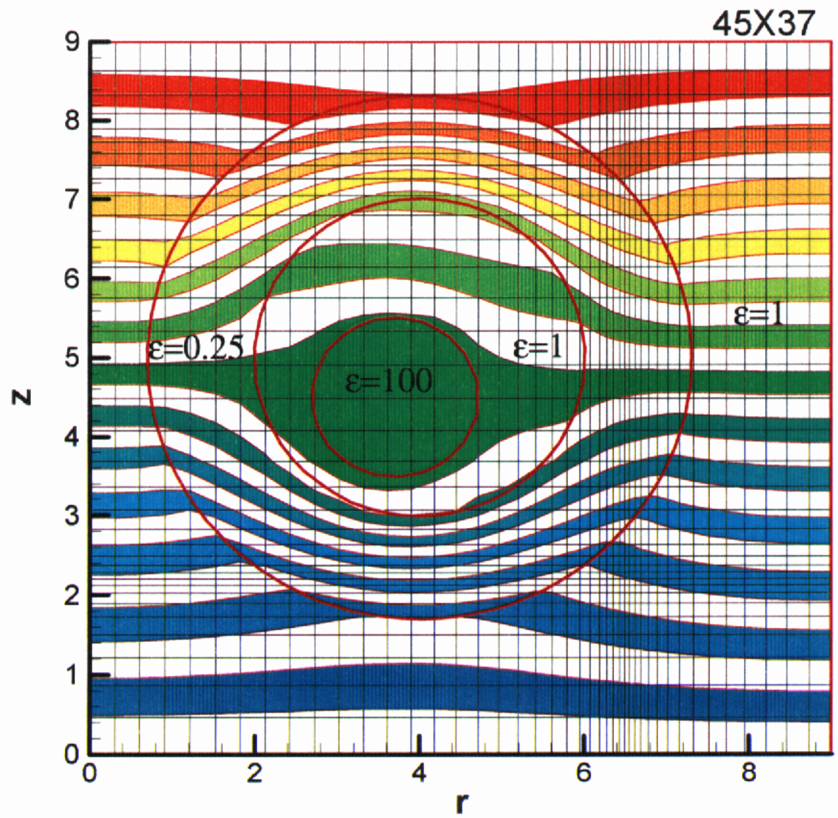
$$\left[\frac{\epsilon_{i+1}\delta_p S_p + (\epsilon S - \sigma\delta_p)(\Delta_p - \delta_p)}{\Delta_{ED}} - \epsilon S\right] / \delta_p \Delta - \epsilon_m (S - S_m)/\Delta_m \Delta = -\rho$$

where $\epsilon\Delta_{ED} \equiv [\epsilon_{i+1}\delta_p + \epsilon(\Delta_p - \delta_p)]$ The equations simplifies leading to coefficients

$$\begin{array}{lcc} & i & i+1 \\ & ------ & ------ \\ S_p: & \epsilon_{i+1}/\Delta_{ED}\Delta & \epsilon_p'/\Delta_p'\Delta' \\ S & -[\epsilon_{i+1}/\Delta_{ED} + \epsilon_m/\Delta_m]/\Delta & -\left[\epsilon_p'/\Delta_p' + \epsilon_{i-1}'/\Delta_{ED}'\right]/\Delta' \\ S_m: & \epsilon_m/\Delta_m\Delta & \epsilon_{i-1}'/\Delta_{ED}'\Delta' \\ rhs: & -\rho + \frac{\sigma(\Delta_p - \delta_p)}{\Delta_{ED}\Delta} & -\rho' + \frac{\sigma'\left(\Delta_m' - \delta_m'\right)}{\Delta_{ED}'\Delta'} \end{array} \tag{4}$$

where $\epsilon'\Delta_{ED}' \equiv \left[\epsilon_{i-1}'\delta_m' + \epsilon'\left(\Delta_m' - \delta_m'\right)\right]$ and the other $'$ coefficients have analogous definitions when viewed from the $i+1$ point. The full, unmodified coefficients are solved in the interior of the dielectric The generalization to 2D is quite straightforward, leading to the introduction of a simple cosine of the intercept of the PWLS into the effective surface charge. The construction of the gradients is more tedious but also straightforward and is described in Ref 1

An example of this dielectric condition is shown the contour plot below. S lines tend to be compressed in regions with $\epsilon < 1$ and excluded from regions where ϵ is large Dielectric boundary conditions on the ECB interfaces. On the left and right are Neumann zero boundary conditions while Dirichlet boundary conditions are used on the top (S=constant) and bottom (S=0). Notice that the solution is better than the contour plotter; the solution is computed on the mesh points using the precise location of the boundary intercept; the contour plotter interpolates only *between* mesh points



Additionally, since ECB does not change the matrix structure but only the element values near a structure, any linear system solver may be used without modification.

References

1) D.W. Hewett, "The Embedded Curved Boundary Method for Orthogonal Meshes", J. Comp. Phys. (in press, 1998).

2) M.A. Lambert, G.L. Rodrigue, and D.W. Hewett, "A Parallel DSDADI Method for Solution of the Steady State Diffusion Equation", (accepted Parallel Computing), 1997.

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